THE INVENTION OF SPACE

by Professor F. M. CORNFORD

LAST YEAR (1934) the President of the British Association opened his address on a despondent note. In the last half-century, he remarked, the main edifice of science had grown almost beyond recognition, as whole armies of labourers added wing after wing, storey upon storey. "Yet the theoretical physicist must admit that his own department looks like nothing so much as a building which has been brought down in ruins by a succession of earthquake shocks."

Laymen have heard the rumbling of these earthquakes. We understand that they have irreparably damaged the framework of space and time, which formerly served like a skeleton of steel girders to support the structure and determine the outline of the external world. We are more or less disconcerted by the news. When I was taught geometry, geometry and Euclid were synonymous terms; and it never occurred to me to doubt that I lived and moved in Euclidean space, extending, quite obviously, in all its three dimensions, without limit. I suspect that, if we look into our minds, all but a few accomplished mathematicians will find the old framework of space and time still unshaken. Common sense lags a good way behind the reasonings of revolutionary thinkers. We have not yet readjusted our perspective and redrawn our map to accommodate such statements as these, which follow in the President's address:

Neither space nor time is found to exist in its own right, but only as a way of cutting up something more comprehensive—

the space-time continuum. Thus we find that space and time cannot be classified as realities of nature, and the generalized theory of relativity shows that the same is true of their product, the space-time continuum. This can be crumpled and twisted and warped as much as we please without becoming one whit less true to nature—which, of course, can only mean that it is not itself part of nature. Space and time, and also their space-time product, fall into their places as mere mental frameworks of our own construction.

So what we took for the steel structure of the universe turns out to be less like steel than india-rubber; and the india-rubber itself exists only as an arbitrary figment of the human brain. It will be some time yet before common sense assimilates this doctrine and begins to think easily in terms of its concepts.

I am not now concerned with a problem that might perplex the simple mind: How can a ray of light be sure of travelling all round a space that can be twisted and crumpled at the mathematician's pleasure? And
200 if it cannot, what becomes of the astronomer's hope that, if he can only wait long enough, he will see the back of his own head through "a sufficiently powerful telescope"? But there remains a question of interest to those who still care to know something of our inheritance from the past. How did the illusion of the steel framework, as an external fact, come to be imposed upon common sense? If the infinite extent of three-dimensional space is no more than a construction of the human brain and only one of many possible alternatives, all equally agreeable to nature, when and by whom was it constructed? Did the Euclidean era, from which we are now emerging, stretch back, with no definable limit, through all recorded history into the darkness of the Stone Age? Was the geometry set

forth by Euclid in the ordered steps of logical deduction simply an explicit formulation of what common sense, from the dawn of human life, had always implicitly conceived? That might naturally be assumed so long as Euclidean space was taken either as a given fact of external nature or as an equally given fact in the constitution of our own minds, needing only to be discovered and displayed in rational argument. But now that it is being replaced by an arbitrary fabric of non-existent india-rubber, the assumption may be questioned.

By whom, then, was the framework created? Professor Eddington's suspicions fall upon Euclid himself. "The only thing," he writes, "that can be urged against spherical space is that more than twenty centuries ago a certain Greek published a set of axioms which (inferentially) stated that spherical space is impossible. He had, perhaps, more excuse, but no more reason, for his statement than those who repeat it
to-day."* Euclid was teaching at Alexandria round 201
about 300 B.C. Probably he had been trained in Athens by Plato's pupils at the Academy. But he was, in the main, only codifying a geometry which had been built up piecemeal in scattered theorems by Greek mathematicians of the preceding three centuries. The work was begun in the sixth century by Thales and Pythagoras, the first parents of the two parallel traditions of philosophy. I seek to show that the belief in infinite space as a physical fact can be traced back to the Greek philosophers of the three centuries between Thales and Euclid, but no farther. Granted that we are dealing here with a fabrication of human brains, the brains in question were active

* *The Expanding Universe* (1933), p. 40.

between 600 and 300 B.C. Their figment came to be finally imposed on science in the Euclidean era now ending, and to be so deeply ingrained in common sense that we shall find it hard to assimilate the india-rubber substitute.

If this is true, we are concerned with a product of Western civilization in its Hellenic phase. It would help my thesis if I were in a position to show that the Indians or the Chinese, before Western science spread all round the globe, had some different scheme of conception. This seems to me likely, but ignorance confines my argument to the history of Western thought and to the documents that I can read and hope to understand.

Within these limits we start from the known fact that Euclidean geometry was constructed, from beginning to end, by the Greeks of those three centuries. How did they arrive at the notion of that familiar space in which straight lines travel on for ever
202 farther and farther from their starting-point? If, as we are told, such a space does not exist, of its own right, in nature, the construction does not come immediately from observation. Nor can we fall back on the assumption that the mathematicians were simply formulating the implicit conceptions of immemorial common sense; for the ordinary man, no more than the philosophic geometer, could observe what was not there. The inference is that the belief in the infinite extent of space, implied by this geometry, was not implicit in the mind of the Homeric or pre-Homeric Greek. There was a pre-Euclidean common sense, whose conception of the world in space had to be transformed into the Euclidean conception, just as our Euclidean common sense has now to

be transformed into the post-Euclidean scheme of relativity.

The evidence for that earlier transformation is to be found in the philosophic literature of our three centuries. As present experience shows, a readjustment of this order cannot be made suddenly; for several generations the old ideas may persist, while common sense lags behind the fresh discoveries of the most advanced minds. In antiquity knowledge spread slowly. When Democritus came to Athens, which lies about 220 miles from his native city, he complained that no one had ever heard his name. A revolution of thought such as may now take one or two generations, might well take a couple of centuries in ancient Greece. The literature of the great creative period preserves abundant traces of the resistance offered by pre-Euclidean common sense to the then revolutionary doctrine of infinite space.

For our purpose the essential property of Euclidean
203 space is that it had no centre and no circumference. In its full abstraction, as conceived by the mathematician, it was an immeasurable blank field, on which the mind could describe all the perfect figures of geometry, but which had no inherent shape of its own. For the physicist it was the frame of the material universe, partly occupied by visible or tangible bodies, whose number and extent were again without definable limit.

Now this physical frame figures in the atomistic systems of antiquity as the Void. The illimitable inane of Lucretius is taken from Epicurus, the contemporary of Euclid. Epicurus took it from the earlier atomists, Leucippus and Democritus, who were at work in the second half of the fifth century. It was

these atomists who maintained the existence of an unlimited Void, as a fact in nature. I would suggest that, in so doing, they were endowing the abstract space implied in Greek geometry with physical existence.

As I read the story, what happened was briefly this. As geometry developed, mathematicians were unconsciously led to postulate the infinite space required for the construction of their geometrical figures—that space in which parallel straight lines can be produced "indefinitely" without meeting or reverting to their starting-point. In the sixth and fifth centuries no distinction was yet drawn between the space demanded by the theorems of geometry and the space which frames the physical world. We know from Aristotle that the earlier Pythagoreans did not even distinguish the solid figures of geometry from the bodies we daily see and handle. Hence the considerations which led mathematicians to recognize infinite space in their science simultaneously led some physicists to recognize an unlimited Void in nature. These were the atomists, whose system was the final outcome of a tradition inspired by Pythagorean mathematics. The atomists broke down the ancient boundaries of the universe and set before mankind, for the first time, the abhorrent and really unimaginable picture of a limitless Void.

If this summary account is correct, the space framework finally accepted by physical science in the Euclidean era is simply the Void of Lucretius. We asked how that framework came to be constructed and imposed upon common sense. It remains to substantiate in more detail the answer suggested: that it was constructed by the reasoning of Greek geometers and imposed by the atomists.

Consider first the progress of geometry towards its final form in the thirteen books of Euclid. We find it presented there as a rigid chain of logical deduction, starting from a number of indemonstrable premisses—definitions, postulates, common notions—and proceeding to more and more complex theorems, in which every step is guaranteed by some previous conclusion. But this form gives no picture of the process by which the various parts of the structure were first discovered. The proposition that the square on the hypotenuse of a right-angled triangle is equal to the squares on the other two sides is said to be due to Pythagoras himself, one of the founders of the whole science. There is no reason to doubt the tradition, and I hope it is also true that Pythagoras sacrificed an ox in the joy of his discovery. Now in Euclid this theorem stands as the last but one in the first book, preceded by forty-six prior propositions and by all the ultimate premisses. But Pythagoras lighted upon it as an isolated truth, and had no idea that he ought to demonstrate forty-six other propositions before he would be warranted in sacrificing his ox. Geometry, in fact, was discovered piecemeal by many independent minds, who attacked particular problems or hit upon particular theorems without co-ordinating their results. So might an unexplored country be mapped by a number of surveyors, each working outwards from a different point and covering as large an area as he could manage. Later some geographer might piece these fragments together. He would find gaps needing to be filled in, and only then would he see the outlines of the country as a whole. In geometry this work of co-ordination was partly done in the fourth century by Plato's colleagues and pupils, and it was trium-

phantly completed by Euclid. The task involved working backwards, as well as forwards, along the chain of deduction. Among the last things to be established would be those which stand first in the final presentation, the irreducible collection of premisses on which the whole structure depends. In the *Republic* Plato complains that the examination of first premisses had been neglected. It was left for his own school to undertake the task and to carry it, as they supposed, to completion.

I suggest—though I cannot directly prove this—that geometrical space itself may be compared to the outline of a country revealed for the first time to the co-ordinating geographer. It was not realized from the first that the figures employed in the scattered theorems demanded a space of infinite extent. If we suppose this discovery to date from about the middle of the fifth century, then, since the theorems seemed to be established beyond doubt, we can understand why the space they implied was accepted by the atomists as the framework of reality.

I am led to this conjecture by the history of the Void—a curious history traceable through the philosophic writings of our period. I have suggested that the infinite Void is simply Euclidean space credited, as a matter of course, with physical existence. But its existence was maintained by the atomists only in the teeth of very considerable opposition. At the end of our period it was still denied by Aristotle; and his immense authority, fortified by ecclesiastical prejudice, held atomism at bay until physics began to move forward again at the Renaissance. Here, however, we are concerned with the question why the infinite Void met with opposition in the fifth and fourth centuries.

The answer lies, at least partly, in the resistance of pre-Euclidean common sense, persisting in the minds of most philosophers. It follows that the space of Euclidean geometry was not the accepted framework of nature before the geometers had mapped it out.

Our first glimpse of the Void in philosophic literature we owe to a passage in Aristotle's *Physics*, recording a feature of the primitive Pythagorean cosmology:

> The Pythagoreans too asserted that Void exists and that it enters the Heaven itself, which, as it were, breathes in from the boundless a sort of breath* which is at the same time the Void. This keeps things apart, as if it constituted a sort of separation or distinction between things that are next to each other. This holds primarily in the case of numbers; for it is the Void that distinguishes their nature (*Phys.*, IV, 6, 213*b*, 23).

The very obscurity of this statement is witness enough to the archaic character of the system described, which must go back to the sixth century. We are to imagine a spherical universe called "the Heaven," a living creature, whose breath is drawn in from the boundless air enveloping it outside. The important point is that "the Void" is another name for this air or breath. As Aristotle notes in the neighbouring context, we still speak of a vessel as empty when in fact it is filled with air. Within the Heaven, the function of this air or vacancy is to keep apart the solid bodies we see and to give them room to move in. Thanks to these vacant intervals, body or matter is not one solid immovable block, but a

* Reading πνεῦμά τι for πνεύματος: πνεῦμα Heidel: πνεῦμά τε Diels. The Oxford translat on retains πνεύματος and renders: "and that it enters the heaven itself, which as it were inhales it, from the infinite air" (ἐκ τοῦ ἀπείρου πνεύματος). The variation does not affect my argument.

plurality of discrete things that can move about. The physical picture is not hard to imagine.

What baffles us at first sight is Aristotle's last sentence: "This holds primarily in the case of numbers; for it is the Void that distinguishes their nature." The Pythagoreans represented numbers by patterns of dots or pebbles or counters, arranged in squares, triangles, and other figures, as on our dice and dominoes. Hence we still speak of "square numbers," "cubes," and so forth. The Void which distinguishes their nature is the blank intervals between these units, or the gaps separating the terms in the series of natural integers.* Moreover, since these units were disposed in regular geometrical shapes, the Void is also the blank field (χώρα) marked off by the boundary lines of geometrical figures.† Under this aspect the Void was the space of geometry.

To our minds it is barely possible to confuse the empty gaps between terms in a series of numbers
208 with the physical air or vacancy that keeps solid bodies apart, or even with geometrical space. But we are dealing here with the most primitive form of atomism, older by perhaps half a century than the atomism of Leucippus. These Pythagoreans simply identified the units of number with geometrical points having position in actual space and an indivisible magnitude. They held that physical bodies actually *are* numbers—a number being defined as a plurality of units. Visible and tangible bodies are built up of

* Cf. Simplicius, *Phys.*, 652, 4 (on this passage): "For what else separates 1 from 2 or 2 from 3 but the void, there being no existence between them?"

† Proclus (on *Euclid*, I, Def. xiii, p. 136, Friedlein) remarks that the geometrical term "boundary" (ὅρος) belongs of right to the primitive "land-measurement" (γεωμετρία), whereby areas of land were measured and their boundaries kept distinct.

these monads, which are the units of arithmetic, the points of geometry, and the atoms of body, all at once. The monads are preserved in discrete plurality by intervals of vacancy—gaps between terms in a numerical series, space between the boundaries of geometrical figures, air between the atoms of body and between the surfaces of different bodies.

The essential function of the Void in this system is to keep things apart inside the spherical Heaven and give them room to move in. For that purpose the living Heaven breathes in the vacant air that laps it round. Obviously this internal Void (as we may call it) raises no question of infinite extension. The arguments in defence of the Void, reviewed and criticized by Aristotle in the context, are arguments for the internal Void. It was alleged that bodies could not move without empty spaces to move into; that bodies could not otherwise expand or contract; and that the growth of animals could occur only if the substances they eat could find vacant spaces to occupy.* The whole 209
controversy about the existence of such vacant intervals could be carried on without raising the question of an infinite extent of Void outside the Heaven.

But there is something outside the Heaven; beyond the circumference is the enveloping air which the world breathes. So far there is no occasion for an infinite extent of Void or air outside. The life on our planet is sustained by an envelope of air a few miles in depth; we do not need that air shall spread to the limits of the galaxy and beyond it for ever. Must we suppose that the air round the Pythagorean Heaven was strictly unlimited?

When Aristotle speaks of the Heaven inhaling its

* *Physics*, IV, 6; cf. Lucretius, i, 329 ff.

breath "from the boundless" (ἐκ τοῦ ἀπείρου), some would take that word as meaning just that unlimited extent which belongs to the Void in later atomism. Were that so, my thesis would fall to the ground; because here, at the threshold of Greek cosmology, we should find infinite space already established in physical existence. Everything hangs on the meaning and implications of "the boundless" in sixth-century Greek. "The boundless" figures in the still older Milesian systems of Anaximander and his successor Anaximenes. Anaximander taught that our cosmos was formed of materials drawn from a "boundless" body which at all times encompasses the world. Anaximenes identified this body with air—that very air which the Pythagorean Heaven breathes. Scholars have debated whether the word ἄπειρον meant "indefinite in quality" or "unlimited in extent," or both. Some who hold that quantity, rather than quality, is in question have assumed that "boundless" implies
210 the infinite and shapeless extent of Euclidean space or the Lucretian Void. But the word by no means excludes the idea of shape. On the contrary, it is frequently and specially used of circular or spherical shape, because on the circumference of the circle or the sphere there is no beginning or end, no boundary separating one part from another. A scholium on the *Iliad* (Ξ, 200), discussing the phrase "the limits of the earth," quotes Porphyry to the following effect. The circumference of the circle and the surface of the sphere are the only figures that can be called in every way uniform. Hence the ancients with good reason described the circle and the sphere as "boundless." Thus Aristophanes has the phrase "Wearing a boundless bronze finger-ring," meaning a ring having

no juncture, no limit as beginning or end. Rings having a bezel with an inset gem are not "boundless," as not being uniform. Aeschylus again speaks of women standing round an altar "in a boundless company," meaning the circular arrangement. Euripides calls the seamless tunic "a boundless texture," and speaks of the ether as boundless because it is round (κυκλοτερής) and embraces the earth in its arms.*

The last quotation is specially significant. Porphyry takes "boundless ether" to mean the round encompassing sky, which the speaker in Euripides identifies with Zeus or god. So Anaximander described his "boundless," encompassing the world, as "the divine"; and Empedocles calls his divine universe "a rounded sphere altogether boundless."† Thus the word "boundless" in itself affords no reason to suppose that the enveloping air of Anaximenes or Pythagoras was of infinite extent. There is, on the contrary, some ground for thinking that it actually implied spherical shape.

This interpretation becomes still more probable 211
when we consider the slightly later system of Parmenides. The whole of being, he declares, "since it has a furthest limit, is complete on every side, like the mass of a rounded sphere, equally poised from the centre in every direction." We naturally ask, what is outside this finite sphere of being? Parmenides does not raise that question; apparently it did not occur

* Ar. *Danaids*, fr. 250, δακτύλιον χαλκοῦν φέρων ἄπειρον (cf. Aristotle, *Phys.*, 207*a*, 2, τοὺς δακτυλίους ἀπείρους λέγουσι τοὺς μὴ ἔχοντας σφενδόνην). Aesch. fr. 279, ἐν λόχῳ ἀπείρονι. Eurip. fr. 279, ὕφασμα ἄπειρον, fr. 941, ὁρᾶς τὸν ὑψοῦ τόνδ' ἄπειρον αἰθέρα | καὶ γῆν περιξ ἔχουθ' ὑγραῖς ἐν ἀγκάλαις; | τοῦτον νόμιζε Ζῆνα, τόνδ' ἡγοῦ θεόν.

† Emped., fr. 28, πάμπαν ἀπείρων Σφαῖρος κυκλοτερής. Ar. *Met.*, 1074, *b*1, "The ancients from the most remote ages have handed down to posterity a mythical tradition that these (the heavenly bodies) are gods and that the divine encompasses the whole of nature."

to him that such a question could be asked. On the other hand, we are left in no doubt as to the answer by what he says about the internal Void of the Pythagoreans. He flatly denied its existence. The Void, to his mind, is simply "nothing," and what is nothing can have no existence. The internal Void, as we saw, was to keep things apart and to provide room for motion. Parmenides accepted the consequence: since the Void, being nothing, cannot exist, a plurality of separate things and motion is impossible. Reality becomes one solid immovable block. The appearance of plurality and motion must be somehow illusory. Now, if nothing cannot exist inside the world, neither can it exist outside. If we do ask the question he ignores, the only possible answer is: Outside the One Being there can be neither something (for all being is inside) nor nothing (for nothing cannot exist or even be conceived).* If we find this answer baffling, the fault lies in our own Euclidean preconception that space must extend without limit; therefore, beyond a finite sphere containing all being there must be an endless waste of nothing. The difficulty vanishes when we realize that, at Parmenides' date, no one had seen any reason why there should be an infinity of unoccupied space.

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It appears, then, that in these earliest cosmologies the universe of being was finite and spherical, with no endless stretch of emptiness beyond. Space had the form of that which filled space—the form of a sphere with centre and circumference. The point in dispute was, whether the sphere was entirely compact

* Cf. Plato, *Theaet.* 180 E. Parmenides declared that the One Being is at rest within its own limits, "having no room in which it moves." There is no vacant space outside, in which it could move about.

with body or there were vacant intervals inside. Parmenides' denial of these intervals shows that the distinction between air, which is something, and the true Void, which is nothing, was beginning to be drawn. The true Void had its origin in the mathematical Void invoked by the Pythagoreans to separate the units of number. As mathematics became more independent of physics, the confusion of intervals between numbers with the air keeping apart physical bodies could not long persist. So the true Void came to be distinguished from air. Anaxagoras, in the fifth century, demonstrated by experiments with "empty" wine-skins and waterclocks that the spaces we call empty are really filled with air, which is something, since it resists pressure. By proving the substantial existence of air he thought he was disproving the existence of any true Void.

After Parmenides, the first task of physics was to restore the possibility of a plurality of things and of motion in space. Atomism was revived in a less questionable form by Leucippus and Democritus. They met Parmenides' denial of the Void with a bold reply. "I admit," said Leucippus, "that the Void—sheer emptiness—is 'nothing' or 'not-being.' All the same, this nothing does exist no less than the something, the compact being, we call body." Thus the internal Void was reasserted, no longer as air, but as the true Void. By this time, if my hypothesis is correct, geometers were realizing that their science called for a space of unlimited extent. Geometry, moreover, was detaching itself from arithmetic. It was now denied that space is made up of points that could be identified with the units of number separated by intervals of emptiness, the discontinuous arithmeti-

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cal Void of the Pythagoreans. Geometrical space was seen to be continuous, not a pattern of empty gaps interrupted by solid things; it penetrates the solids that partly occupy its single continuous medium. At the same time the theorems of geometry were seen to require that parallel straight lines shall travel on for ever through this medium without meeting or returning upon themselves. The atomists now take the revolutionary step of ascribing to a physical Void, outside the visible Heaven, the infinite extent of this geometrical space.* Atoms, they held, must be illimitable in number and therefore demand an unlimited extent of space.

The consequences were far more outrageous to pre-Euclidean common sense than we, who have assimilated infinite space, can easily realize. Space was now robbed of its circumference, and therefore of any centre. The immemorial claim of the Earth to be at the centre of the universe was impiously denied. The
214 Earth might still be at the centre of our finite world; but our world has been cut adrift in a limitless waste that has no centre. And now, for the first time,† appears the consequent belief in innumerable worlds (*cosmoi*) scattered over endless space. At all times some are coming into being, others passing away. These other worlds were not the same as any stars,

* Simplicius, *Phys.*, 648, 11, "These asserted the actual existence of an interval between bodies which prevents their being continuous, as Democritus and Leucippus held, who declared that there is a void, not only inside the cosmos, but also outside—a thing which clearly will not be 'place,' but something with an independent existence."

† In the minds of most modern writers the whole question of infinite space has been prejudiced by Burnet's attribution of the doctrine of innumerable worlds to Anaximander and other pre-atomists. I have argued (*Classical Quarterly*, XXVIII, 1934, pp. 1 ff.) that the evidence he relied on is worthless.

or clusters of stars, that we can see: all the stars belong to our world. They were what might now be called "island universes," whose existence, entirely beyond the range of observation, was asserted on *a priori* grounds as a reasonable probability. Thus Metrodorus of Chios argued: "That there should be only one world in the infinite would be as strange as that a single ear of corn should grow in a large plain." Epicurus pointed out that no limited number of worlds could exhaust the unlimited supply of atoms, and Lucretius followed. This doctrine could not arise until the ancient boundaries of spherical space had been broken down and the belief in its strictly infinite extent had deprived space of any centre for our Earth to occupy.

Thus atomism created the picture of illimitable and shapeless vacancy with its sparse population of unnumbered worlds. Once drawn, the picture could never be forgotten so long as men could read Lucre-
215 tius. It remained to be accepted by the physicists who revived atomism in modern times. Meanwhile its truth was strenuously denied by philosophers who clung to the spherical image of pre-Euclidean common sense. Commentators on the *Timaeus* have doubted whether Plato admits even the internal Void. Aristotle denied any Void, whether internal or external. He demonstrates that there cannot be more than one world, and that the encompassing Heaven is necessarily spherical. Outside the Heaven, he says, there can be "no place or void or time." The Void had been defined as that in which the presence of body, though not actual, is possible. Body cannot exist outside the Heaven. Therefore there is no external Void (*de caelo* i, 9).

It only remained to point out that the space of geometry, if it really required infinite extension, was not, after all, the same thing as physical space. For Plato the objects and truths of mathematics belong to an intelligible realm; the physical world is no more than an imperfect copy or reflection. Geometrical space could be disposed of as an object of thought, not of the senses, or as in some way imaginary. The geometer may claim that his straight lines can be produced "indefinitely"; but no one can actually draw a line of infinite length. All the mathematician needs, says Aristotle, is a finite line produced as far as he pleases. Aristotle's theory, Dr. Ross remarks, "is here somewhat obscure. He holds strongly that the physical world is a sphere of finite size. The mathematician cannot have a straight line greater than the diameter of this sphere present to him in sensation, and the meaning must be that he is free to *imagine* such a line if he chooses, and if he can."*

216 So tenacious was the resistance of pre-Euclidean common sense. The Greek mind recoiled in horror from the boundless vacancy its own reasonings had conjured into existence. It would be fantastic to suggest that a sound instinct held it back. But listen once more to Sir James Jeans:†

> Are there any limits at all to the extent of space?
>
> Even a generation ago I think most scientists would have answered this question in the negative. They would have argued that space could be limited only by the presence of something which is not space. We, or rather our imaginations, could only be prevented from journeying for ever through space by running up against a wall of something different from space. And, hard though it may be to imagine space extending for ever, it is far harder to imagine a barrier of something different from space which could prevent our imaginations from passing into a further space beyond.

* *Aristotle* (1923), p. 85. Ar. *Phys.*, 207*b*, 27.
† *The Universe Around Us*, p. 70.

So the Euclidean has argued ever since Archytas, Plato's contemporary, reasoned thus:

> If I am at the extremity of the heaven of the fixed stars, can I stretch outwards my hand or staff? It is absurd to suppose that I could not; and if I can, what is outside must be either body or space. We may then in the same way get to the outside of that again, and so on; and if there is always a new place to which the staff may be held out, this clearly involves extension without limit (Eudemus, frag. 30).

So, too, Lucretius:

> If for the moment all existing space be held to be bounded, supposing a man runs forward to its outside borders and stands on the utmost verge and then throws a winged javelin, do you choose that when hurled with vigorous force it shall fly to 217
> a distance, or do you decide that something can get in its way and stop it? for you must admit and adopt one of the two suppositions; either of which shuts you out from all escape and compels you to grant that the universe stretches without end (1, 968, trans. Munro).

Some two thousand years after Archytas, John Locke repeats his argument:

> If body be not supposed infinite, which I think no one will affirm, I would ask, Whether, if God placed a man at the extremity of corporeal beings, he could not stretch out his hand beyond his body? If he could, then he would put his arm where there was before space without body. . . . If he could not stretch out his hand, it must be because of some external hindrance . . . and then I ask, Whether that which hinders

his hand be substance or accident, something or nothing? . . . I would fain meet with that thinking man, that can in his thoughts set any bounds to space, more than he can to duration; or by thinking hope to arrive at the end of either (*Essay*, ii, 13, 21).

But this argument, Sir James Jeans continues, is not a sound one.

For instance, the earth's surface is of limited extent, but there is no barrier which prevents us from travelling on and on as far as we please. A traveller who did not understand that the earth's surface is spherical would naturally expect that longer and longer journeys would for ever open up new tracts of country awaiting exploration. Yet, as we know, he would necessarily be reduced in time to repeating his own tracks. As a result of its curvature, the earth's surface, although unlimited, is finite in extent. Through his theory of relativity, Einstein claims to have established that space also, although unlimited, is finite in extent. The total volume of space in the universe is of finite amount, just as the surface of the earth is of finite amount, and for the same reason; both bend back on themselves and close up. . . . As a consequence of space bending back into itself, a projectile or a ray of light can travel on for ever without going outside space into something which is not space, and yet it cannot go on for ever without repeating its own tracks. For this reason it is probable that light can travel round the whole of space and return to its starting-point, so that if we pointed a sufficiently powerful telescope in the right direction in the midnight sky, we should see the sun and its neighbours in space by light which had made the circuit of the universe.

This post-Euclidean finite but unbounded space takes us back to the pre-Euclidean finite but boundless sphere of Anaximander, Parmenides, and Empedocles. These philosophers did not know as much

mathematics as Einstein; but they had the advantage over Newton in knowing much less mathematics than Euclid. They had not been misled by geometry into projecting its infinite space into the external world under the name of the Void. The Euclidean era thus presents itself as a period of aberration, in which common sense was reluctantly lured away from the position that it has now, with no less reluctance, to regain. The whirligig of Time has brought in his revenges upon the impious assailants of spherical Space.

Tantum irreligio potuit suadere malorum.

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The subject of this essay lies only towards the margin of that field of study to which Gilbert Murray has brought gifts of insight and imagination not often combined with exquisite scholarship. The better, perhaps, its claim to stand as a tribute to the breadth of interest of one who has been willing to lay aside his chosen pursuits for the service of mankind.